Oh, where does the centroid go?

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Prospectus Up to scale there is only one catenary, $f(x) = \cosh(x)$. One of its interesting geometric qualities is that if $[a, b]$ is in the domain of f, then the geometric centroid of the arc-length of the graph of f between $x = a$ and $x = b$ lies directly above the centroid of the area under the graph of f, $x \in [a, b]$ and the x-axis.

Question 1: Does this property characterize the catenary? If not, what other curves are so characterized? Can the curve be the boundary for a smooth, compact object or is it necessarily unbounded.

Question 2: What happens if we stipulate that the centroids of the arc and area are two points on various geometric figures such as an isosceles triangle? Can we isolate interesting (well-known) curves?

Question 3: The centroid of plane figures has to do with the first moment about the coordinate axes. What about higher order moments?

References

[1] V. Coll and M. Harrison, *Two generalizations of a property of the catenary.* To appear.

[2] E. Parker, *A property characterizing the catenary*, Mathematics Magazine **83 No.1** (February 2010) 63–64.